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A Study on Neutrosophic Soft Set and Neutrosophic Hypersoft Set

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Abstract

This paper mainly intends to verify whether the basic laws of set theory apply in the Neutrosophic Soft Set (NSS). The laws examined in this paper for NSS include Commutative law, Associative law, Distributive law, Involution law, Idempotent law, and Negation law (law of contradiction and law of excluded middle). It has also been illustrated with sufficient examples. In addition, extending that to the hypersoft set, we have proved if H is a Neutrosophic hypersoft Subgroup (NHSSG) of a group G and N is a normal subgroup of G, then HN is a NHSSG of G.

Keywords: Neutrosophic set, Neutrosophic soft set, Complement set, Neutrosophic hypersoft set.

1 | Introduction

Zadeh [1] dealt with the most needed and complicated case of vagueness. Fuzzy theory is an effective tool used to describe vague concepts. Fuzzy theory is indicated with the membership function in the interval [0, 1]. Represents full non belongingness, and 1 represents full belongingness. To deal with membership as well as non-membership functions, Atanassov [2] developed Intuitionistic Fuzzy Sets (IFSs). Yager [3] extended IFSs to the Pythagorean Fuzzy set. After that, the Neutrosophy concept was introduced, with three components: 1) truthfulness, 2) indeterminacy, and 3) falseness was introduced by Smarandache [4]. Molodtsov [5] initiated the soft set theory concepts. Rosenfeld [6] extended classical group theory concepts to fuzzy sets, IFS, and soft sets.

Kumar Maji [7] established the Neutrosophic Soft Set (NSS), and he introduced basic operations of soft sets like a union of two soft sets, the intersection of soft sets, equality of soft sets, complement of soft sets, subsets

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and supersets, absolute of Soft set. Also, he verified De Morgan's law and other useful results. Ali et al. [8] proposed new notions of union, intersection, and complement of soft sets. Babitha and Sunil [9] introduced soft set relations and discussed some concepts like partition, composition, functions, etc. Maji et al. [10] used soft sets for the first time in decision-making problems. Chen et al. [11] explain the soft set parameterization reduction and apply it to decision-making problems. Said and Smarandache [12] introduced the intuitionistic NSS and its operations. Later, many researchers produced their research using Neutrosophy concepts in different mathematical areas. Smarandache [13] introduced a hypersoft set as a generalization of soft sets. Recently, Smarandache [14] introduced new types of soft sets, such as the hypersoft set, indeterm soft set, indeterm hypersoft set, and tree soft set [15]. In practical real life, we are forced to face indeterministic challenges to compete and survive in the modern engineering world. So, in this paper, the fundamental properties of NSS are studied with suitable examples. Also, we have proved that HN is a Neutrosophic hypersoft Subgroup (NHSSG) of G if N is normal and H is NHSSG.

2 | Preliminaries

In this section, we recall the basic notions of some important definitions.

Definition 1 ([4]). Neutrosophic set: "let U be the universe of discourse then the Neutrosophic set A is an object having the form A = {< x: TA(x), IA(x), FA(x) >, x ∈U}, where the functions T, I, F: U →] - 0,1+[define respectively the degree of membership, the degree of indeterminacy, and the degree of non-membership of the element x ∈U to the set A with the condition.

$$-0 \leq \mathrm{TA(x)} + \mathrm{IA(x)} + \mathrm{FA(x)} \leq 3+$$

Definition 2 ([16]). A binary operation ∗: [0, 1] × [0, 1] → [0, 1] is a continuous t-norm if it satisfies the following conditions: (i) ∗ is commutative and associative. (ii) ∗ is continuous, (iii) $a * 1 = 1 * a = a$, for all $a \in [0, 1]$, (iv) $a * b \leq c * d$ if $a \leq c$, $b \leq$ d, with $a, b, c, d \in [0, 1]$. A few examples of continuous t-norm are $a * b = ab$, $a * b = min\ (a, b)$, $a * b = max\ (a + b - 1, 0)$.

Definition 3 ([16]). A binary operation ⋄ : [0, 1] × [0, 1] → [0, 1] is continuous t – conorm (s - norm) if it satisfies the following conditions: (i) ⋄ is commutative and associative, (ii) ⋄ is continuous, (iii) $a \diamond 0 = 0 \diamond a = a$, for all $a \in [0, 1]$, (iv) $a \diamond b \leq c \diamond d$ if $a \leq c, b \leq d$ with $a, b, c, d \in [0, 1]$. A few examples of continuous s-norm are $a \diamond b = a + b - ab$, $a \diamond b =$ max (a, b), $a \diamond b =$ min $(a + b, 1)$. For all $a \in [0, 1]$.

If $a * a = a$ and $a \diamond a = a$, then ∗ is called an idempotent t-norm, and ⋄ is called an idempotent s-norm.

Definition 4 ([16]). Let U be an initial universe set and E be a set of parameters. Let (U) denote the set of all NSs of U. Then, a NSS N over U is a set defined by a set-valued function fN representing a mapping fN: $E \rightarrow P(U)$ where fN is called approximate function of the NSS N. In other words, the NSS is a parameterized family of some elements of the set (U), and therefore, it can be written as a set of ordered pairs, $\mathrm{N} = \{(\mathrm{e}, \{< \mathrm{x}, \mathrm{T_{fN(e)}(x)}, \mathrm{I_{fN(e)}(x)}, \mathrm{F_{fN(e)}(x)} >: \mathrm{x} \in \mathrm{U}\}): \mathrm{e} \in \mathrm{E}\}$where $\mathrm{T_{fN(e)}(x)}, \mathrm{I_{fN(e)}(x,\ F_{fN(e)}(x} \in [0,1]$, respectively, called the truth-membership, indeterminacy-membership, falsity-membership function of $fN(e)$. Since the supremum of each T, I, F is 1, the inequality $0 \leq \mathrm{T_{fN(e)}(x)} + \mathrm{I_{fN(e)}(x)} + \mathrm{F_{fN(e)}(x)} \leq 3$ is obvious.

Definition 5 ([16]). The complement of a NSS N is denoted by N' and is defined as: $\mathrm{N'} = \{(\mathrm{e}, \{< \mathrm{x}, \mathrm{F_{fN(e)}(x)}, 1 - \mathrm{I_{fN(e)}(x)}, \mathrm{T_{fN(e)}(x)} >: \mathrm{x} \in \mathrm{U}\}): \mathrm{e} \in \mathrm{E}\}$.

Definition 6 ([17]). (Empty or null NSS with respect to a parameter.) A NSS N over the universe U is termed to be an empty or null NSS with respect to the parameter e if

$$\mathrm{T_{fN(e)}(x)} = 0, \mathrm{I_{fN(e)}(x)} = 0, \mathrm{F_{fN(e)}(x)} = 0.$$

Definition 7 ([18]). A subgroup N of the group G is normal in G iff gNg-1 ∈ N for all g ∈G and n ∈N.

Definition 8 ([19]). Let U be a universe of discourse, P (U) the power set of U. Let a1, a2, …, an, for n ≥ 1, be n distinct attributes, whose corresponding attribute values are respectively the sets A1, A2, …, An, with Ai ∩ Aj = Φ, for i ≠ j, and i, j in {1, 2, …, n}. Then the pair (F, A1 × A2 × … × An), where F: A1 × A2 × … × An→ P (U), is called a hypersoft set over U.

3 | Fundamental Properties of Nss

Theorem 1. Let N_1, N_2 and N_3 be three NSS over the common universe (U, E). Then, the following properties hold.

Commutative law

$$N_1 \cup N_2 = N_2 \cup N_1,$$

$$N_1 \cap N_2 = N_2 \cap N_1.$$

Proof: Let N_1 and N_2 be two NSSs. Since $\Diamond$ & $*$ is commutative, then,

$$N_1 \cup N_2 = T_{fN_1(e)}(x) \Diamond T_{fN_2(e)}(x) = T_{fN_2(e)}(x) \Diamond T_{fN_1(e)}(x),$$
$$I_{fN_1(e)}(x) * I_{fN_2(e)}(x) = I_{fN_2(e)}(x) * I_{fN_1(e)}(x),$$
$$F_{fN_1(e)}(x) * F_{fN_2(e)}(x) = F_{fN_2(e)}(x) * F_{fN_1(e)}(x),$$
$$= N_2 \cup N_1$$
$$\Rightarrow N_1 \cup N_2 = N_2 \cup N_1.$$

Therefore, Commutative law is proved.

Similarly, we can prove $N_1 \cap N_2 = N_2 \cup N_1$.

Associative law

$$(N_1 \cup N_2) \cup N_3 = N_1 \cup (N_2 \cup N_3),$$

$$(N_1 \cap N_2) \cap N_3 = N_1 \cap (N_2 \cap N_3).$$

Proof: Let N_1 and N_2 be two NSSs. Since $\Diamond$ & $*$ is commutative and associative, we have

$$(N_1 \cup N_2) \cup N_3 =$$
$$\left[T_{fN_1(e)}(x) \Diamond T_{fN_2(e)}(x)\right] \Diamond T_{fN_3(e)}(x) = T_{fN_1(e)}(x) \Diamond \left[T_{fN_2(e)}(x) \Diamond T_{fN_3(e)}(x)\right],$$
$$\left[I_{fN_1(e)}(x) * I_{fN_2(e)}(x)\right] * I_{fN_3(e)}(x) = I_{fN_1(e)}(x) * \left[I_{fN_2(e)}(x) * I_{fN_3(e)}(x)\right],$$
$$\left[F_{fN_1(e)}(x) * F_{fN_2(e)}(x)\right] * F_{fN_3(e)}(x) = F_{fN_1(e)}(x) * \left[F_{fN_2(e)}(x) * F_{fN_3(e)}(x)\right],$$
$$[\because * \text{ is associative}],$$
$$= N_1 \cup (N_2 \cup N_3).$$

Associative property has been proved.

Similarly, $(N_1 \cap N_2) \cap N_3 = N_1 \cap (N_2 \cap N_3)$.

Distributive law

$$N_1 \cup (N_2 \cap N_3) = (N_1 \cup N_2) \cap (N_1 \cup N_3),$$

$$N_1 \cap (N_2 \cup N_3) = (N_1 \cap N_2) \cup (N_1 \cap N_3),$$

Proof: Let N_1 and N_2 be two NSSs. Since $\Diamond$ & $*$ is commutative and associative, then,

$$N_1 \cup (N_2 \cap N_3)$$
$$= T_{fN_1(e)}(x) \Diamond \left[T_{fN_2(e)}(x) * T_{fN_3(e)}(x)\right]$$
$$= \left[T_{fN_1(e)}(x) \Diamond T_{fN_2(e)}(x)\right] * \left[T_{fN_1(e)}(x) \Diamond T_{fN_3(e)}(x)\right].$$

Next,

$I_{fN_1(e)}(x) \lozenge [I_{fN_2(e)}(x) * I_{fN_3(e)}(x)] = [I_{fN_1(e)}(x) \lozenge I_{fN_2(e)}(x)] * [I_{fN_1(e)}(x) \lozenge I_{fN_3(e)}(x)]$.

Similarly,

$F_{fN_1(e)}(x) \lozenge [F_{fN_2(e)}(x) * F_{fN_3(e)}(x)] = [F_{fN_1(e)}(x) \lozenge F_{fN_2(e)}(x)] * [F_{fN_1(e)}(x) \lozenge F_{fN_3(e)}(x)]$
$= (N_1 \cup N_2) \cap (N_1 \cup N_3)$.

Similarly, we can prove $N_1 \cap (N_2 \cup N_3) = (N_1 \cap N_2) \cup (N_1 \cap N_3)$.

Involution law

i.e. $(N')' = N$.

Proof: Let N be NSS and N' be the complement of N.

$N = \{(e, \{< x, T_{fN(e)}(x), I_{fN(e)}(x), F_{fN(e)}(x) >: x \in U\}): e \in E\}$,
$N' = \{(e, \{< x, F_{fN(e)}(x), 1 - I_{fN(e)}(x), T_{fN(e)}(x) >: x \in U\}): e \in E\}$,
$(N')' = \{(e, \{< x, T_{fN(e)}(x), I_{fN(e)}(x), F_{fN(e)}(x) >: x \in U\}): e \in E\} = N$.

This implies Involution law.

Idempotent law

$N \cup N = N$,
$N \cap N = N$.

Proof: Let N be NSSs. Since $\lozenge$ & $*$ is commutative and associative, we have

$N = \{(e, \{< x, T_{fN(e)}(x), I_{fN(e)}(x), F_{fN(e)}(x) >: x \in U\}): e \in E\}$,
$N \cup N \Rightarrow T_{fN(e)}(x) \lozenge T_{fN(e)}(x) = T_{fN(e)}(x)$, $[\because a \lozenge a = a]$,
$I_{fN(e)}(x) * I_{fN(e)}(x) = I_{fN(e)}(x)$, $[\because a * a = a]$,
$F_{fN(e)}(x) * F_{fN(e)}(x) = F_{fN(e)}(x)$, $[\because a * a = a]$,
$= \{(e, \{< x, T_{fN(e)}(x), I_{fN(e)}(x), F_{fN(e)}(x) >: x \in U\}): e \in E\}$
$= N$.

Similarly, we can prove $N \cap N = N$.

Negation law

Let N denote NSS over the common universe (U, E) and N' be the complement of N, then the Negation law doesn't hold.

i.e. $N \cap N' \neq \emptyset$.

$N \cup N' \neq \emptyset$.

Proof: Let N be NSS and N' be the complement of N.

Since $\lozenge$ & $*$ is commutative and associative, we have

$N = \{(e, \{< x, T_{fN(e)}(x), I_{fN(e)}(x), F_{fN(e)}(x) >: x \in U\}): e \in E\}$,
$N' = \{(e, \{< x, F_{fN(e)}(x), 1 - I_{fN(e)}(x), T_{fN(e)}(x) >: x \in U\}): e \in E\}$,
$N \cap N' = T_{fN(e)}(x) * F_{fN(e)}(x) = \emptyset$,
$I_{fN(e)}(x) \lozenge 1 - I_{fN(e)}(x) = 1$, $[\because a \lozenge b = \min(a+b,1) = 1]$,
$F_{fN(e)}(x) \lozenge T_{fN(e)}(x) = 1$, $[\min(F+T,1) = \min(U, 1) = 1]$,
$\Rightarrow N \cap N' \neq \emptyset$.

Similarly,

$N \cup N' \Rightarrow$
$T_{fN(e)}(x) \lozenge F_{fN(e)}(x) = U,$
$I_{fN_1(e)}(x) * 1 - I_{fN(e)}(x) = \emptyset,$ $[\because a * b = \max(a+b-1, 0) = 0 = \emptyset],$
$F_{fN(e)}(x) * T_{fN(e)}(x) = \emptyset,$ $[\because a * b = \max(a+b-1, 0) = \emptyset],$
$\Rightarrow N \cap N' \neq \emptyset \ \& N \cup N' \neq U.$

Hence, the Negation law doesn't hold.

Identity law

$N \cup \emptyset = N,$

$N \cap \emptyset = \emptyset.$

Proof: Let N be NSS. Since $\lozenge$ & $*$ is commutative and associative, we have

$N \cup \emptyset \Rightarrow T_{fN(e)}(x) \lozenge \emptyset = T_{fN(e)}(x),$
$[a \lozenge b = \max(a, b)],$
$I_{fN(e)}(x) * \emptyset = I_{fN(e)}(x),$
$[a * b = \min(a, b)],$
$F_{fN(e)}(x) * \emptyset = F_{fN(e)}(x),$ $[a * b = \min(a, b)].$

This implies $N \cup \emptyset = N$.

Similarly,

$N \cap \emptyset = \emptyset \Rightarrow$ Identity law holds.

De Morgan's law

$(N_1 \cup N_2)' = N_1' \cap N_2',$
$(N_1 \cap N_2)' = N_1' \cup N_2'.$

Proof: Let N_1 and N_2 be two NSS. Since $\lozenge$ & $*$ is commutative and associative, we have

$$(N_1 \cup N_2)' = \begin{bmatrix} T_{fN_1(e)}(x) \lozenge T_{fN_2(e)}(x) \\ I_{fN_1(e)}(x) * I_{fN_2(e)}(x) \\ F_{fN_1(e)}(x) * F_{fN_2(e)}(x) \end{bmatrix}'$$
$= F_{fN_1(e)}(x) * F_{fN_2(e)}(x),$
$1 - (I_{fN_1(e)}(x) \lozenge I_{fN_2(e)}(x)),$
$T_{fN_1(e)}(x) \lozenge T_{fN_2(e)}(x).$
$N_1' = \{(e, \{< x, F_{N_1(e)}(x), 1 - I_{fN_1(e)}(x), T_{fN_1(e)}(x) > : x \in U\}) : e \in E\},$
$N_2' = \{(e, \{< x, F_{N_2(e)}(x), 1 - I_{fN_2(e)}(x), T_{fN_2(e)}(x) > : x \in U\}) : e \in E\},$
$N_1' \cap N_2' = F_{fN_1(e)}(x) * F_{N_2(e)}(x),$
$1 - (I_{fN_1(e)}(x) \lozenge I_{fN_2(e)}(x)),$
$T_{fN_1(e)}(x) \lozenge T_{fN_2(e)}(x),$
This proves $(N_1 \cup N_2)' = N_1' \cap N_2'$.

Similarly, $(N_1 \cap N_2)' = N_1' \cup N_2'$ can be proved.

Absorption law

$N_1 \cup (N_1 \cap N_2) = N_1,$

$N_1 \cap (N_1 \cup N_2) = N_1.$

Proof: Let N_1 and N_2 be two NSSs. Since $\lozenge$ & $*$ is commutative and associative, we have

$N_1 \cup (N_1 \cap N_2)$
$= T_{fN_1(e)}(x) \lozenge (T_{fN_1(e)}(x) * T_{fN_2(e)}(x))$,
[if $a \subset b$, $a * b = \min(a, b) = a$],
$= T_{fN_1(e)}(x) \lozenge T_{fN_1(e)}(x)$
$= T_{fN_1(e)}(x)$, $[a \lozenge b = \max(a, b)]$.

Similarly,

$I_{fN_1(e)}(x) \lozenge [I_{fN_1(e)}(x) * I_{fN_2(e)}(x)]$
$= I_{fN_1(e)}(x) \lozenge I_{fN_1(e)}(x)$, $\because$ Since $a \subset b, a * b = \min(a, b)$

$= I_{fN_1(e)}(x)$.

Also

$F_{fN_1(e)}(x) \lozenge [F_{fN_1(e)}(x) * F_{fN_2(e)}(x)]$
$= F_{fN_1(e)}(x) \lozenge F_{fN_1(e)}(x)$ $\because$ Since $F_{fN_1} \subset F_{fN_2}$
$= F_{fN_1(e)}(x)$

$\Rightarrow N_1 \cup (N_1 \cap N_2) = N_1$.

Similarly, we can prove $N_1 \cap (N_1 \cup N_2) = N_1$.

4 | Illustration

Consider $U = \{u_1, u_2, u_3\}$ and $E = \{e_1, e_2, e_3\}$ N_1, N_2 and N_3 be three NSSs, then prove the fundamental laws in NSS.

Table 1. N_1.

	$f_{N_1}(e_1)$	$f_{N_1}(e_2)$	$f_{N_1}(e_3)$
u_1	(0.4, 0.3, 0.2)	(0.1, 0.5, 0.6)	(0.7, 0.3, 0.2)
u_2	(0.2, 0.6, 0.3)	(0.6, 0.5, 0.4)	(0.3, 0.6, 0.4)
u_3	(0.4, 0.2, 0.6)	(0.4, 0.3, 0.1)	(0.6, 0.2, 0.1)

Table 2. N_2.

	$f_{N_2}(e_1)$	$f_{N_2}(e_2)$	$f_{N_2}(e_3)$
u_1	(0.2, 0.6, 0.1)	(0.4, 0.2, 0.6)	(0.1, 0.2, 0.3)
u_2	(0.4, 0.3, 0.2)	(0.2, 0.4, 0.5)	(0.6, 0.3, 0.2)
u_3	(0.1, 0.2, 0.3)	(0.2, 0.3, 0.2)	(0.3, 0.2, 0.1)

Table 3. N_3.

	$f_{N_3}(e_1)$	$f_{N_3}(e_2)$	$f_{N_3}(e_3)$
u_1	(0.5, 0.4, 0.3)	(0.2, 0.6, 0.1)	(0.5, 0.4, 0.2)
u_2	(0.6, 0.2, 0.1)	(0.4, 0.6, 0.5)	(0.4, 0.3, 0.2)
u_3	(0.1, 0.2, 0.5)	(0.6, 0.2, 0.1)	(0.3, 0.2, 0.1)

Solution:

Commutative law

To prove $N_1 \cup N_2 = N_2 \cup N_1$.

Table 4. $N_1 \cup N_2$.

	$f_{N_1 \cup N_2}(e_1)$	$f_{N_1 \cup N_2}(e_2)$	$f_{N_1 \cup N_2}(e_3)$
u_1	(0.4, 0.3, 0.1)	(0.4, 0.2, 0.6)	(0.7, 0.2, 0.2)
u_2	(0.4, 0.3, 0.2)	(0.6, 0.4, 0.4)	(0.6, 0.3, 0.2)
u_3	(0.4, 0.2, 0.3)	(0.4, 0.3, 0.1)	(0.6, 0.2, 0.1)

Table 5. $N_2 \cup N_1$.

	$f_{N_2 \cup N_1}(e_1)$	$f_{N_2 \cup N_1}(e_2)$	$f_{N_2 \cup N_1}(e_3)$
u_1	(0.4, 0.3, 0.1)	(0.4, 02, 0.6)	(0.7, 0.2, 0.2)
u_2	(0.4, 0.3, 0.2)	(0.6, 0.4, 0.4)	(0.6, 0.3, 0.2)
u_3	(0.4, 0.2, 0.3)	(0.4, 0.3, 0.1)	(0.6, 0.2, 0.1)

$N_1 \cup N_2 = N_2 \cup N_1$.

Similarly,

$N_1 \cap N_2 = N_2 \cap N_1$

=> Commutative law has been proved.

Associative law

To prove $N_1 \cup (N_2 \cup N_3) = (N_1 \cup N_2) \cup N_3$.

Table 6. $N_1 \cup (N_2 \cup N_3)$.

	$f_A(e_1)$	$f_A(e_2)$	$f_A(e_3)$
u_1	(0.5, 0.3, 0.1)	(0.4, 0.2, 0.1)	(0.7, 0.2, 0.2)
u_2	(0.6, 0.2, 0.1)	(0.6, 0.4, 0.4)	(0.6, 0.3, 0.2)
u_3	(0.4, 0.2, 0.3)	(0.6, 0.2, 0.1)	(0.6, 0.2, 0.1)

Table 7. $N_1 \cup N_2) \cup N_3 = B$.

	$f_B(e_1)$	$f_B(e_2)$	$f_B(e_3)$
u_1	(0.5, 0.3, 0.1)	(0.4, 0.2, 0.1)	(0.7, 0.2, 0.2)
u_2	(0.6, 0.2, 0.1)	(0.6, 0.4, 0.4)	(0.6, 0.3, 0.2)
u_3	(0.4, 0.2, 0.3)	(0.6, 0.2, 0.1)	(0.6, 0.2, 0.1)

Associative law verified.

Similarly, we can prove $N_1 \cap (N_2 \cap N_3) = (N_1 \cap N_2) \cap N_3$.

De Morgan's law

$(N_1 \cup N_2)' = N_1' \cap N_2'$.

Table 8. $(N_1 \cup N_2)'$.

	$f_p(e_1)$	$f_p(e_2)$	$f_p(e_3)$
u_1	(0.1, 0.7, 0.4)	(0.6, 0.8, 0.4)	(0.2, 0.8, 0.7)
u_2	(0.2, 0.7, 0.4)	(0.4, 0.6, 0.6)	(0.2, 0.7, 0.6)
u_3	(0.3, 0.8, 0.4)	(0.1, 0.7, 0.4)	(0.1, 0.8, 0.6)

We refer to these tables as number (1).

Table 9. N_1'.

	$f_{N_1'}(e_1)$	$f_{N_2'}(e_2)$	$f_{N_3'}(e_3)$
u_1	(0.2, 0.7, 0.4)	(0.6, 0.5, 0.1)	(0.2, 0.7, 0.7)
u_2	(0.3, 0.4, 0.2)	(0.4, 0.5, 0.6)	(0.4, 0.4, 0.3)
u_3	(0.6, 0.8, 0.4)	(0.1, 0.7, 0.4)	(0.1, 0.8, 0.6)

Table 10. N_2'.

	$f_{N_1'}(e_1)$	$f_{N_2'}(e_2)$	$f_{N_3'}(e_3)$
u_1	(0.1, 0.4, 0.2)	(0.6, 0.8, 0.4)	(0.3, 0.8, 0.1)
u_2	(0.2, 0.7, 0.4)	(0.5, 0.6, 0.2)	(0.2, 0.7, 0.6)
u_3	(0.3, 0.8, 0.1)	(0.2, 0.7, 0.2)	(0.1, 0.8, 0.3)

Table 11. $N_1' \cap N_2'$.

	$f_{N_q'}(e_1)$	$f_{N_q'}(e_2)$	$f_{N_q'}(e_3)$
u_1	(0.1, 0.7, 0.4)	(0.6, 0.8, 0.4)	(0.2, 0.8, 0.7)
u_2	(0.2, 0.7, 0.4)	(0.4, 0.6, 0.6)	(0.2, 0.7, 0.6)
u_3	(0.3, 0.8, 0.4)	(0.1, 0.7, 0.4)	(0.1, 0.8, 0.6)

We refer to these tables as number (2).

(1)= (2)

Similarly, we can prove

$(N_1 \cup N_2)' = N_1' \cap N_2'$.

=> De Morgan law verified.

Idempotent Law

$N_1 \cup N_1 = N_1$.

Table 12. $N_1 \cup N_1$.

	$f_{N_1 \cup N_1}(e_1)$	$f_{N_1 \cup N_1}(e_2)$	$f_{N_1 \cup N_1}(e_3)$
u_1	(0.4, 0.3, 0.2)	(0.1, 0.5, 0.6)	(0.7, 0.3, 0.2)
u_2	(0.2, 0.6, 0.3)	(0.6, 0.5, 0.4)	(0.3, 0.6, 0.4)
u_3	(0.4, 0.2, 0.6)	(0.4, 0.3, 0.1)	(0.6, 0.2, 0.1)

$= N_1$

Similarly, $N_1 \cap N_1 = N_1$ can be proved

=> Idempotent law verified.

Absorption law

$N_1 \cup (N_1 \cap N_2) = N_1$.

Table 13. $(N_1 \cap N_2)$.

	$f_{(N_1 \cap N_2)}(e_1)$	$f_{(N_1 \cap N_2)}(e_2)$	$f_{(N_1 \cap N_2)}(e_3)$
u_1	(0.2, 0.6, 0.2)	(0.1, 0.5, 0.6)	(0.1, 0.3, 0.3)
u_2	(0.2, 0.6, 0.3)	(0.2, 0.5, 0.5)	(0.3, 0.6, 0.4)
u_3	(0.1, 0.2, 0.6)	(0.2, 0.3, 0.2)	(0.3, 0.2, 0.1)

Table 14. $N_1 \cup (N_1 \cap N_2)$.

	$f_x(e_1)$	$f_x(e_2)$	$f_x(e_3)$
u_1	(0.4, 0.3, 0.2)	(0.1, 0.5, 0.6)	(0.7, 0.3, 0.2)
u_2	(0.2, 0.6, 0.3)	(0.6, 0.5, 0.4)	(0.3, 0.6, 0.4)
u_3	(0.4, 0.2, 0.6)	(0.4, 0.3, 0.1)	(0.6, 0.2, 0.1)

$N_1 \cup (N_1 \cap N_2) = N_1$.

=> Absorption law has been verified.

Distributive law

$N_1 \cup (N_2 \cap N_3) = (N_1 \cup N_2) \cap (N_1 \cup N_3)$.

Table 15. $(N_2 \cap N_3)$.

	$f_{(N_2 \cap N_3)}(e_1)$	$f_{(N_2 \cap N_3)}(e_2)$	$f_{(N_2 \cap N_3)}(e_3)$
u_1	(0.2, 0.6, 0.3)	(0.2, 0.6, 0.6)	(0.1, 0.4, 0.3)
u_2	(0.4, 0.3, 0.2)	(0.2, 0.6, 0.5)	(0.4, 0.3, 0.2)
u_3	(0.1, 0.2, 0.5)	(0.2, 0.3, 0.2)	(0.3, 0.2, 0.1)

Table 16. $N_1 \cup (N_2 \cap N_3)$.

	$f_s(e_1)$	$f_s(e_2)$	$f_s(e_3)$
u_1	(0.4, 0.3, 0.2)	(0.2, 0.5, 0.6)	(0.7, 0.3, 0.2)
u_2	(0.4, 0.3, 0.2)	(0.6, 0.5, 0.4)	(0.4, 0.3, 0.2)
u_3	(0.4, 0.2, 0.5)	(0.4, 0.3, 0.1)	(0.6, 0.2, 0.1)

We refer to these tables as number (3).

Table 17. $N_1 \cup N_2$).

	$f_{N_1 \cup N_2}(e_1)$	$f_{N_1 \cup N_2}(e_2)$	$f_{N_1 \cup N_2}(e_3)$
u_1	(0.4, 0.3, 0.1)	(0.4, 0.2, 0.6)	(0.7, 0.2, 0.2)
u_2	(0.4, 0.3, 0.2)	(0.6, 0.4, 0.4)	(0.6, 0.3, 0.2)
u_3	(0.4, 0.2, 0.3)	(0.4, 0.3, 0.1)	(0.6, 0.2, 0.1)

Table 18. $N_1 \cup N_3$).

	$f_{N_1 \cup N_3}(e_1)$	$f_{N_1 \cup N_3}(e_2)$	$f_{N_1 \cup N_3}(e_3)$
u_1	(0.5, 0.3, 0.2)	(0.2, 0.5, 0.6)	(0.7, 0.3, 0.2)
u_2	(0.6, 0.2, 0.1)	(0.6, 0.5, 0.4)	(0.4, 0.3, 0.2)
u_3	(0.4, 0.2, 0.5)	(0.6, 0.2, 0.1)	(0.6, 0.2, 0.1)

Table 19. $(N_1 \cup N_2) \cap (N_1 \cup N_3)$.

	$f_t(e_1)$	$f_t(e_2)$	$f_t(e_3)$
u_1	(0.4, 0.3, 0.2)	(0.2, 0.5, 0.6)	(0.7, 0.3, 0.2)
u_2	(0.4, 0.3, 0.2)	(0.6, 0.5, 0.4)	(0.4, 0.3, 0.2)
u_3	(0.4, 0.2, 0.5)	(0.4, 0.3, 0.1)	(0.6, 0.2, 0.1)

We refer to these tables as number (4).

(3) = (4)

Similarly, we can prove, $N_1 \cap (N_2 \cup N_3) = (N_1 \cap N_2) \cup (N_1 \cap N_3)$

=> Distributive law has been proved.

Negation law

$N \cap N' = \emptyset, N \cup N' = \emptyset.$

Table 20. N_1'.

	$f_{N_1'}(e_1)$	$f_{N_1'}(e_2)$	$f_{N_1'}(e_3)$
u_1	(0.2, 0.7, 0.4)	(0.6, 0.5, 0.1)	(0.2, 0.7, 0.7)
u_2	(0.3, 0.4, 0.2)	(0.4, 0.5, 0.6)	(0.4, 0.4, 0.3)
u_3	(0.6, 0.8, 0.4)	(0.1, 0.7, 0.4)	(0.1, 0.8, 0.6)

Table 21. $N_1 \cap N_1'$.

	$f_{N_1 \cap N_1'}(e_1)$	$f_{N_1 \cap N_1'}(e_2)$	$f_{N_1 \cap N_1'}(e_3)$
u_1	(0.2, 0.7, 0.4)	(0.1, 0.5, 0.6)	(0.2, 0.7, 0.7)
u_2	(0.2, 0.6, 0.3)	(0.4, 0.5, 0.6)	(0.3, 0.6, 0.4)
u_3	(0.4, 0.8, 0.6)	(0.1, 0.7, 0.4)	(0.1, 0.8, 0.6)

$\neq \emptyset$

Table 22. $N_1 \cup N_1'$.

	$f_{N_1 \cup N_1'}(e_1)$	$f_{N_1 \cup N_1'}(e_2)$	$f_{N_1 \cup N_1'}(e_3)$
u_1	(0.4, 0.3, 0.2)	(0.6, 0.5, 0.1)	(0.7, 0.3, 0.2)
u_2	(0.3, 0.4, 0.2)	(0.6, 0.5, 0.4)	(0.4, 0.4, 0.3)
u_3	(0.6, 0.2, 0.4)	(0.4, 0.3, 0.1)	(0.6, 0.2, 0.1)

$\neq U$

=> Hence, the Negation law doesn't law hold.

Involution law

$(N_1')' = N_1$.

Table 23. $(N_1')'$.

	$f_{(N_1')'}(e_1)$	$f_{(N_1')'}(e_2)$	$f_{(N_1')'}(e_3)$
u_1	(0.4, 0.3, 0.2)	(0.1, 0.5, 0.6)	(0.7, 0.3, 0.2)
u_2	(0.2, 0.6, 0.3)	(0.6, 0.5, 0.4)	(0.3, 0.6, 0.4)
u_3	(0.4, 0.2, 0.6)	(0.4, 0.3, 0.1)	(0.6, 0.2, 0.1)

$= N_1$

=> Involution law has been proved.

Theorem 2. If H is a NHSSG of a group G and N is a normal Subgroup of G, then HN is a NHSSG of G.

Proof: to prove HN is a NHSSG of G, it is enough if we prove that HN=NH.

Let x ∈ HN, Then x=hn where h ∈H and n ∈N. Therefore x ∈ hN.

Since N is normal, hN=Nh.

Therefore, x ∈ Nh. Hence $x=n_1h$ where $n_1 \in N$.

Therefore x ∈ NH. Hence, HN ⊆ HN. Similarly, NH ⊆HN.

This implies HN=NH.

Hence, HN is a NHSSG of G.

5 | Conclusion

The fundamental properties of set theory have been studied in NSS. Commutative, Associative, Distributive, De Morgan's law, Involution law, Idempotent law, and Absorption law have been proved in NSS. Negation law does not hold in NSS. All these laws are illustrated with suitable examples, and a theorem in the NHSSG has been proved.

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Conflicts of Interest

The authors declare no conflict of interest.

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